Selection and Sorting of Heterogeneous Firms Through Competitive Pressures

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Introduction

Competitive Pressures on Heterogeneous Firms

Main Questions: How do more ***competitive pressures***, due to entry of new firms, caused by lower ***entry cost*** or larger ***market size***, affect firms with different productivity?

- Selection of firms
- Distribution of firm size (in revenue, profit and employment), Distribution of markup and pass-through rates, etc.
- Sorting of firms across markets with different market sizes

Existing Monopolistic Competition Models with Heterogenous Firms

- Melitz (2003): under **CES** Demand System (DS)
  - MC firms sell their products at an exogenous & common markup rate, *unresponsive to competitive pressures*
  - Market size: no effect on distribution of firm types nor their behaviors; All adjustments at *the extensive margin*.
  - Firms' incentive to move across markets with different market sizes independent of firm productivity

  Inconsistent with some evidence for
  - An increase in the production cost leads to less than proportional increase in the price (the pass-through rate < 1)
  - More productive firms have higher markup rates
  - More productive firms have lower pass-through rates
- Melitz-Ottaviano (2008) departs from CES with **Linear Demand System + the outside competitive sector**, which comes with its own restrictions.

This Paper: Melitz under **H.S.A.** Demand System as a framework to study how departing from CES in the direction consistent with the evidence affects the impact of competitive pressures on heterogeneous firms.

Symmetric H.S.A. (Homothetic with a Single Aggregator) DS with Gross Substitutes

Think of a competitive final goods industry generating demand for a continuum of **intermediate inputs** $\omega \in \Omega$, with
CRS production function: $X = X(\mathbf{x})$; $\mathbf{x} = \{x_\omega; \omega \in \Omega\} \Leftrightarrow$ **Unit cost function,** $P = P(\mathbf{p})$; $\mathbf{p} = \{p_\omega; \omega \in \Omega\}$.

Market share of ω depends *solely* on a single variable, its own price normalized by the *common* price aggregator

$$s_\omega \equiv \frac{p_\omega x_\omega}{\mathbf{px}} = \frac{\partial \ln P(\mathbf{p})}{\partial \ln p_\omega} = s\left(\frac{p_\omega}{A(\mathbf{p})}\right), \qquad \text{where} \qquad \int_\Omega s\left(\frac{p_\omega}{A(\mathbf{p})}\right) d\omega \equiv 1.$$

- $s: \mathbb{R}_{++} \to \mathbb{R}_+$: **the market share function**, C^3, decreasing in the **normalized price** $z_\omega \equiv p_\omega / A$ for $s(z_\omega) > 0$ with
  - $\lim_{z \to \bar{z}} s(z) = 0$. If $\bar{z} \equiv \inf\{z > 0 | s(z) = 0\} < \infty$, $\bar{z}A(\mathbf{p})$ is the **choke price**.
- $A = A(\mathbf{p})$: the **common price aggregator** defined implicitly by **the adding-up constraint** $\int_\Omega s(p_\omega/A) d\omega \equiv 1$.
  $A(\mathbf{p})$ linear homogenous in $\mathbf{p}$ for a fixed Ω. A larger Ω reduces $A(\mathbf{p})$.

	CES	$s(z) = \gamma z^{1-\sigma}$;	$\sigma > 1$
Special Cases	**Translog Cost Function**	$s(z) = \gamma \max\{-\ln(z/\bar{z}), 0\}$;	$\bar{z} < \infty$
	Constant Pass Through (CoPaTh)	$s(z) = \gamma \max\left\{\left[\sigma + (1-\sigma) z^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}}, 0\right\}$	$0 < \rho < 1$

As $\rho \nearrow 1$, CoPaTh converges to CES with $\bar{z}(\rho) \equiv (\sigma/(\sigma-1))^{\frac{\rho}{1-\rho}} \to \infty$.

$P(\mathbf{p})$ vs. $A(\mathbf{p})$

Definition: $s_\omega \equiv \frac{\partial \ln P(\mathbf{p})}{\partial \ln p_\omega} = s\left(\frac{p_\omega}{A(\mathbf{p})}\right) = s(z_\omega)$ where $\int_\Omega s\left(\frac{p_\omega}{A(\mathbf{p})}\right) d\omega \equiv 1.$

By differentiating the adding-up constraint,

$$\frac{\partial \ln A(\mathbf{p})}{\partial \ln p_\omega} = \frac{[\zeta(z_\omega) - 1]s(z_\omega)}{\int_\Omega [\zeta(z_{\omega'}) - 1]s(z_{\omega'})d\omega'} \neq s(z_\omega) = \frac{\partial \ln P(\mathbf{p})}{\partial \ln p_\omega}$$

unless $\zeta(z_\omega)$ is constant, where

Price Elasticity Function:

$$\zeta(z) \equiv 1 - \frac{d \ln s(z)}{d \ln z} \equiv 1 - \mathcal{E}_s(z) > 1 \Leftrightarrow s(z) = \gamma \exp\left[\int_{z_0}^{z} \frac{1-\zeta(\xi)}{\xi} d\xi\right]; \ \lim_{z\to\bar{z}} \zeta(z) = \infty, \text{if } \bar{z} < \infty.$$

By integrating the definition,

$$\frac{A(\mathbf{p})}{P(\mathbf{p})} = c \exp\left[\int_\Omega s\left(\frac{p_\omega}{A(\mathbf{p})}\right) \Phi\left(\frac{p_\omega}{A(\mathbf{p})}\right) d\omega\right], \quad \text{where} \quad \Phi(z) \equiv \frac{1}{s(z)} \int_z^{\bar{z}} \frac{s(\xi)}{\xi} d\xi$$

Note: $A(\mathbf{p})/P(\mathbf{p})$ is not constant, unless CES $\Leftrightarrow \zeta(z) = \sigma \Leftrightarrow s(z) = \gamma z^{1-\sigma} \Leftrightarrow \Phi(z) = 1/(\sigma - 1)$.

- ✓ $A(\mathbf{p})$, the inverse measure of *competitive pressures*, captures *cross price effects* in the DS, the reference price for MC firms
- ✓ $P(\mathbf{p})$, the inverse measure of TFP, captures the *productivity effects* of price changes, the reference price for consumers.
- ✓ $\Phi(z)$, the measure of "love for variety." Matsuyama & Ushchev (2023). $\zeta'(\cdot) \gtreqless 0 \implies \Phi'(\cdot) \lesseqgtr 0$; $\Phi'(\cdot) = 0 \Leftrightarrow \zeta'(\cdot) = 0$.

Note: Our 2017 paper proved the integrability = the quasi-concavity of $P(\mathbf{p})$, iff $\zeta(z) \equiv 1 - \frac{d \ln s(z)}{d \ln z} \equiv 1 - \mathcal{E}_s(z) > 0$.

Why H.S.A.

- **Homothetic** (unlike the linear DS and most other commonly used non-CES DSs)
  - a single measure of market size; the demand composition does not matter.
  - isolate the effect of endogenous markup rate from nonhomotheticity
  - straightforward to use it as a building block in multi-sector models with any upper-tier (incl. nonhomothetic) DS

- **Nonparametric** and **flexible** (unlike **CES** and **translog**, which are special cases)
  - can be used to perform robustness-check for CES
  - allow for (but no need to impose)
    - ✓ the choke price,
    - ✓ **Marshall's 2nd law** (Price elasticity is increasing in price) → more productive firms have higher markup rates
    - ✓ *what we call* **the 3rd law** (the rate of increase in the price elasticity is decreasing in price) → more productive firms have lower pass-through rates.

- **Tractable** due to **Single Aggregator** (unlike **Kimball**, which needs two aggregators), a *sufficient statistic* for competitive pressures, which acts like a *magnifier of firm heterogeneity*
  - guarantee the existence & uniqueness of free-entry equilibrium with firm heterogeneity
  - simple to conduct most comparative statics without *parametric* restrictions on demand or productivity distribution.
  - no need to assume zero overhead cost (unlike MO and ACDR)

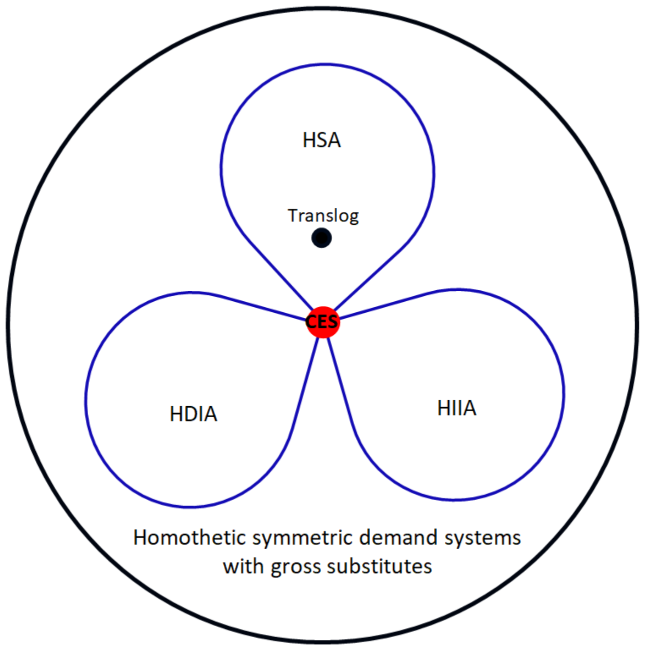
- Defined by **the market share function,** for which data is readily available and easily identifiable.

Three Classes of Homothetic Demand Systems: Matsuyama-Ushchev (2017)

Here we consider a **continuum** of varieties ($\omega \in \Omega$), **gross substitutes**, and **symmetry**

CES	$s_\omega \equiv \frac{\partial \ln P(\mathbf{p})}{\partial \ln p_\omega} = f\left(\frac{p_\omega}{P(\mathbf{p})}\right) \Leftrightarrow s_\omega \propto \left(\frac{p_\omega}{P(\mathbf{p})}\right)^{1-\sigma}$	
H.S.A. (Homotheticity with a Single Aggregator)	$s_\omega = s\left(\frac{p_\omega}{A(\mathbf{p})}\right),$	$\frac{P(\mathbf{p})}{A(\mathbf{p})} \neq c$, unless CES
HDIA (Homotheticity with Direct Implicit Additivity) Kimball is a special case:	$s_\omega = \frac{p_\omega}{P(\mathbf{p})}(\phi')^{-1}\left(\frac{p_\omega}{B(\mathbf{p})}\right),$	$\frac{P(\mathbf{p})}{B(\mathbf{p})} \neq c$, unless CES
HIIA (Homotheticity with Indirect Implicit Additivity)	$s_\omega = \frac{p_\omega}{C(\mathbf{p})}\theta'\left(\frac{p_\omega}{P(\mathbf{p})}\right),$	$\frac{P(\mathbf{p})}{C(\mathbf{p})} \neq c$, unless CES

$\phi(\cdot)$ & $\theta(\cdot)$ are both increasing & concave → $(\phi')^{-1}(\cdot)$ & $\theta'(\cdot)$ positive-valued & decreasing.
$A(\cdot), B(\cdot), C(\cdot)$ all determined by the adding-up constraint.



The 3 classes are pairwise disjoint with the sole exception of CES.

Under HDIA(Kimball) and HIIA, unlike HSA

- Two aggregators needed for the market shares. [One aggregator enough for the price elasticity under all 3 classes.]
- The existence and uniqueness of free-entry equilibrium not guaranteed without some strong restrictions on both productivity distribution and the price elasticity function.

Melitz under HSA: Main Results

- **Existence & Uniqueness of Equilibrium:** straightforward under H.S.A.
- **Melitz under CES:** impacts of entry/overhead costs on the masses of entrants/active firms hinges on the sign of the derivative of the elasticity of the pdf of marginal cost; Pareto is the knife-edge! (new results!)
- **Cross-Sectional Implications:** profits and revenues are always higher among more productive.
  - 2nd Law = incomplete pass-through ⇔ the procompetitive effect ⇔ strategic complementarity in pricing.
  - 2nd (3rd) Law → more productive firms have higher markup (lower pass-through) rates.
  - 2nd & 3rd Laws → *hump-shaped* employment; more productive hire less under high overhead.
- **General Equilibrium Comparative Statics**
  - *Entry cost* ↓: 2nd (3rd) Law → markup rates ↓ (pass-through rates ↑) for all firms.
    profits (revenues) decline faster among less productive → a tougher selection.
  - *Overhead cost* ↓: similar effects when the employment is decreasing in firm productivity.
  - *Market size* ↑: 2nd (3rd) Law → markup rates ↓ (pass-through rates ↑) for all firms.
    profits (revenues) ↑ among more productive; ↓ among less productive.
  - *Due to the composition effect*, these changes may *increase* the average markup rate & the aggregate profit share in spite of 2nd Law and *reduce* the average pass-through in spite of 3rd Law; Pareto is the knife-edge *for entry cost* ↑.
- **Sorting of Heterogeneous Firms** across markets that differ in size: Larger markets→ more competitive pressures.
  - 2nd Law → more (less) productive go into larger (smaller) markets.
  - *Composition effect*, average markup (pass-through) rates can be *higher* (*lower*) in larger and more competitive markets in spite of 2nd (3rd) Law.

(Highly Selective) Literature Review

Non-CES Demand Systems: Matsuyama (2023) for a survey; **H.S.A. Demand System:** Matsuyama-Ushchev (2017)

MC with Heterogeneous Firms: Melitz (2003) and many others: Melitz-Redding (2015) for a survey

MC under non-CES demand systems: Thisse-Ushchev (2018) for a survey

- *Nonhomothetic non-CES:*
  - $U = \int_{\Omega} u(x_{\omega})d\omega$: Dixit-Stiglitz (77), Behrens-Murata (07), ZKPT (12), Mrázová-Neary(17), Dhingra-Morrow (19); ACDR (19)
  - *Linear-demand system with the outside sector:* Ottaviano-Tabuchi-Thisse (2002), Melitz-Ottaviano (2008)
- *Homothetic non-CES:* Feenstra (2003), Kimball (1995), Matsuyama-Ushchev (2020a,b, 2023)
- *H.S.A.* Matsuyama-Ushchev (2022), Kasahara-Sugita (2020), Grossman-Helpman-Lhuiller (2021), Fujiwara-Matsuyama (2022), Baqaee-Fahri-Sangani (2023)

Empirical Evidence: *The 2nd Law:* DeLoecker-Goldberg (14), Burstein-Gopinath (14); *The 3rd Law:* Berman et.al.(12), Amiti et.al.(19), *Market Size Effects:* Campbell-Hopenhayn(05); *Rise of markup:* Autor et.al.(20), DeLoecker et.al.(20)

Selection of Heterogeneous Firms through Competitive Pressures
Melitz-Ottaviano (2008), Baqaee-Fahri-Sangani (2023), Edmond-Midrigan-Xu (2023)

Sorting of Heterogeneous Firms Across Markets:

- *Reduced Form/Partial Equilibrium;* Mrázová-Neary (2019), Nocke (2006)
- *General Equilibrium:* Baldwin-Okubo (2006), Behrens-Duranton-RobertNicoud (2014), Davis-Dingel (2019), Gaubert (2018), Kokovin et.al. (2022)

Log-Super(Sub)modularity: Costinot (2009), Costinot-Vogel (2015)

Structure of the Talk

- Introduction
- Monopolistic Competition under H.S.A.
- Selection of Heterogenous Firms: A Single Market Setting
    - Existence and Uniqueness
    - Cross-Sectional Implications under the 2^{nd} & 3rd Laws
    - Comparative Statics: General Equilibrium Effects
- Sorting of Heterogenous Firms: A Multi-Market Setting
- Appendix: Some Parametric Families of H.S.A.

Monopolistic Competition under H.S.A.

Pricing: Markup & Pass-Through Rates. Taking the value of $A = A(\mathbf{p})$ given, firm ω chooses p_ω.

Lerner Pricing Formula

$$p_\omega\left[1-\frac{1}{\zeta(p_\omega/A)}\right]=\psi_\omega \Longrightarrow \frac{p_\omega}{A}\left[1-\frac{1}{\zeta(p_\omega/A)}\right]=\frac{\psi_\omega}{A},$$

ψ_ω: *firm-specific* (quality-adjusted) marginal cost (in labor, the numeraire)

Under the mild regularity condition, LHS is monotone → firms with the same ψ set the same price → $p_\omega = p_\psi$.

Normalized Price:

$$\frac{p_\psi}{A}\equiv z_\psi = Z\left(\frac{\psi}{A}\right)\in(\psi/A\,,\bar{z}), Z'(\cdot)>0;$$

Price Elasticity: $\zeta(z_\psi)=\zeta\left(Z\left(\frac{\psi}{A}\right)\right)\equiv\sigma\left(\frac{\psi}{A}\right)>1;$ **Markup Rate:** $\mu_\psi\equiv\frac{p_\psi}{\psi}=\frac{\sigma(\psi/A)}{\sigma(\psi/A)-1}\equiv\mu\left(\frac{\psi}{A}\right)>1$

$$\Longrightarrow \frac{1}{\sigma(\psi/A)}+\frac{1}{\mu(\psi/A)}=1 \Leftrightarrow \left[\sigma\left(\frac{\psi}{A}\right)-1\right]\left[\mu\left(\frac{\psi}{A}\right)-1\right]=1$$

Pass-Through Rate:

$$\rho_\psi\equiv\frac{\partial\ln p_\psi}{\partial\ln\psi}=\mathcal{E}_Z\left(\frac{\psi}{A}\right)\equiv\rho\left(\frac{\psi}{A}\right)=1+\mathcal{E}_\mu\left(\frac{\psi}{A}\right)=1-\frac{\mathcal{E}_\sigma(\psi/A)}{\sigma(\psi/A)-1}>0$$

are all functions of the *normalized cost,* ψ/A*, only*; continuously differentiable.

- Market size $L=\mathbf{px}$ affects the pricing behaviors of firms only through its effects on A.
- More competitive pressures, a lower A, act like a magnifier of firm heterogeneity.

Under CES, $\sigma(\cdot)=\sigma;\ \mu(\cdot)=\sigma/(\sigma-1)=\mu;\ \rho(\cdot)=1.$

Revenue, Profit, & Employment

Revenue $R_\psi = s(z_\psi)L = s\left(Z\left(\frac{\psi}{A}\right)\right)L \equiv r\left(\frac{\psi}{A}\right)L \quad \Longrightarrow \quad \mathcal{E}_r\left(\frac{\psi}{A}\right) = -\left[\sigma\left(\frac{\psi}{A}\right) - 1\right]\rho\left(\frac{\psi}{A}\right) < 0$

(Gross) Profit $\Pi_\psi = \frac{r(\psi/A)}{\sigma(\psi/A)}L \equiv \pi\left(\frac{\psi}{A}\right)L \quad \Longrightarrow \quad \mathcal{E}_\pi\left(\frac{\psi}{A}\right) = 1 - \sigma\left(\frac{\psi}{A}\right) < 0$

(Variable) Employment $\psi x_\psi = \frac{r(\psi/A)}{\mu(\psi/A)}L \equiv \ell\left(\frac{\psi}{A}\right)L \quad \Longrightarrow \quad \mathcal{E}_\ell\left(\frac{\psi}{A}\right) = 1 - \sigma\left(\frac{\psi}{A}\right)\rho\left(\frac{\psi}{A}\right) \lesseqgtr 0$

- Revenue $r(\psi/A)L$, profit $\pi(\psi/A)L$, employment $\ell(\psi/A)L$ all functions of ψ/A, multiplied by **market size** L, continuously differentiable under mild regularity conditions.

- Their elasticities $\mathcal{E}_r(\cdot), \mathcal{E}_\pi(\cdot)$ and $\mathcal{E}_\ell(\cdot)$ depend solely on $\sigma(\cdot)$ and $\rho(\cdot)$.
  More competitive pressures, a lower A, act like a magnifier of firm heterogeneity.
  Market size affects the distribution of the profit, revenue and employment across firms only via its effects on A.

Under CES, $r(\cdot)/\pi(\cdot) = \sigma$; $r(\cdot)/\ell(\cdot) = \mu = \sigma/(\sigma-1) \Longrightarrow \mathcal{E}_r(\cdot) = \mathcal{E}_\pi(\cdot) = \mathcal{E}_\ell(\cdot) = 1 - \sigma < 0$.

- Both revenue $r(\psi/A)L$ and profit $\pi(\psi/A)L$ are always strictly decreasing in ψ/A.

- Employment $\ell(\psi/A)L$ may be nonmonotonic in ψ/A.

Selection of Heterogenous Firms: A Single-Market Setting

General Equilibrium: Existence & Uniqueness

As in Melitz, Market size = total labor supply is $L > 0$

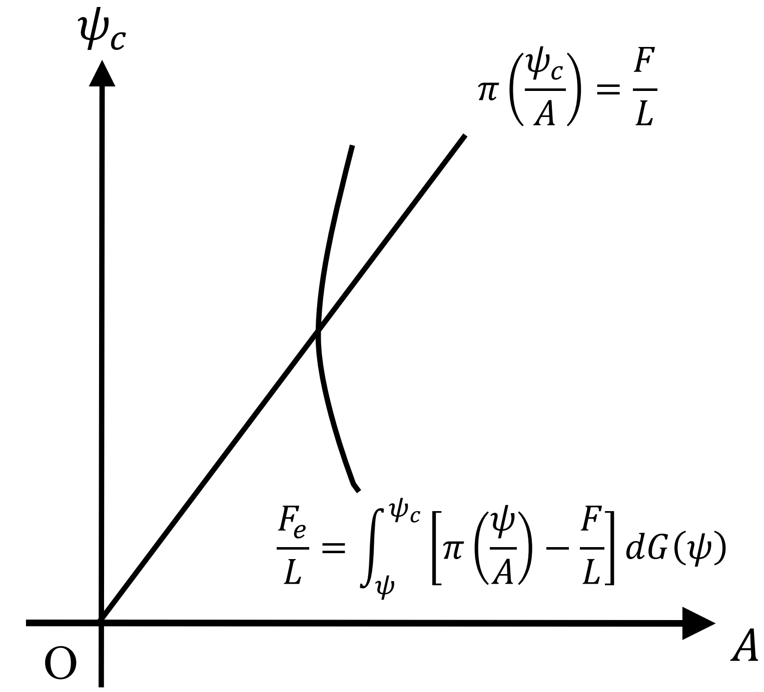
Ex-ante identical firms pay the entry cost $F_e > 0$ to draw $\psi \sim G(\psi)$, cdf whose support, $(\underline{\psi}, \bar{\psi}) \subset (0, \infty)$,

After learning ψ, decide whether to pay the overhead $F > 0$ to stay & produce.

Cutoff Rule: stay if $\psi < \psi_c$; exit if $\psi > \psi_c$, where

$$\max_{\psi_c} \int_{\underline{\psi}}^{\psi_c} \left[\pi\left(\frac{\psi}{A}\right) L - F\right] dG(\psi) \Longrightarrow \pi\left(\frac{\psi_c}{A}\right) L = F$$

positive-sloped, as $A \downarrow$ (more competitive pressures) ⇒ $\psi_c \downarrow$ (tougher selection).
rotate clockwise, as $F/L \uparrow$ (higher overhead/market size) $\Longrightarrow \psi_c/A \downarrow$.



Free Entry Condition:

$$F_e = \int_{\underline{\psi}}^{\psi_c} \left[\pi\left(\frac{\psi}{A}\right) L - F\right] dG(\psi)$$

shift to the left as $F_e \downarrow$ (lower entry cost) $\Longrightarrow A \downarrow$ (more competitive pressures).

$A = A(\mathbf{p})$ and ψ_c: uniquely determined, respond continuously to F_e/L & F/L under mild regularity conditions.
(This proof of unique existence applies also to the Melitz model under CES.)

Equilibrium Mass of Firms With A & ψ_c determined, from **the Adding-up Constraint,**

Mass of Active Firms
= the measure of Ω.

$$MG(\psi_c) = \left[\int_{\underline{\psi}}^{\psi_c} r\left(\frac{\psi}{A}\right)\frac{dG(\psi)}{G(\psi_c)}\right]^{-1} = \left[\int_{\underline{\xi}}^{1} r\left(\frac{\psi_c}{A}\xi\right)d\tilde{G}(\xi;\psi_c)\right]^{-1} > 0$$

where

$$\tilde{G}(\xi;\psi_c) \equiv \frac{G(\psi_c\xi)}{G(\psi_c)}$$

is the cdf of $\xi \equiv \psi/\psi_c$, conditional on $\underline{\xi} \equiv \underline{\psi}/\psi_c < \xi \leq 1$.

Lemma 1: $\mathcal{E}_g'(\psi) < 0 \Longrightarrow \mathcal{E}_G'(\psi) < 0$; $\mathcal{E}_g'(\psi) \geq 0 \Longrightarrow \mathcal{E}_G'(\psi) \geq 0$, with some boundary conditions.
Lemma 2: A lower ψ_c shifts $\tilde{G}(\xi;\psi_c)$ to the right (left) in MLR if $\mathcal{E}_g'(\psi) < (>)0$ and in FSD if $\mathcal{E}_G'(\psi) < (>)0$.

- Some evidence for $\mathcal{E}_g'(\psi) > 0 \Longrightarrow \psi_c \downarrow$ (tougher selection) shifts $\tilde{G}(\xi;\psi_c)$ to the left.
- Pareto-productivity, $G(\psi) = (\psi/\bar{\psi})^\kappa \Longrightarrow \mathcal{E}_g'(\psi) = \mathcal{E}_G'(\psi) = 0 \Longrightarrow \tilde{G}(\xi;\psi_c)$ is independent of ψ_c.
- Fréchet, Weibull, Lognormal; $\mathcal{E}_g'(\psi) < 0 \Longrightarrow \mathcal{E}_G'(\psi) < 0 \Longrightarrow \psi_c \downarrow$ (tougher selection) shifts $\tilde{G}(\xi;\psi_c)$ to the right.

Equilibrium can be solved recursively under H.S.A.!!
Under HDIA/HIIA, one needs to solve the 3 equations simultaneously for 3 variables, ψ_c & the two price aggregates.

Aggregate Labor Cost and Profit Shares and TFP

Notations:

The $w(\cdot)$-weighted average of $f(\cdot)$ among the active firms, $\psi \in \left(\underline{\psi}, \psi_c\right)$	$\mathbb{E}_w(f) \equiv \frac{\int_{\underline{\psi}}^{\psi_c} f\left(\frac{\psi}{A}\right) w\left(\frac{\psi}{A}\right) dG(\psi)}{\int_{\underline{\psi}}^{\psi_c} w\left(\frac{\psi}{A}\right) dG(\psi)}.$
The unweighted average of $f(\cdot)$ among the active firms, $\psi \in \left(\underline{\psi}, \psi_c\right)$	$\mathbb{E}_1(f) \equiv \frac{\int_{\underline{\psi}}^{\psi_c} f\left(\frac{\psi}{A}\right) dG(\psi)}{\int_{\underline{\psi}}^{\psi_c} dG(\psi)}.$

$$\Longrightarrow \mathbb{E}_w\left(\frac{f}{w}\right) = \frac{\mathbb{E}_1(f)}{\mathbb{E}_1(w)} = \left[\mathbb{E}_f\left(\frac{w}{f}\right)\right]^{-1}.$$

Then,

Aggregate TFP	$\ln\left(\frac{X}{L}\right) = \ln\left(\frac{1}{P}\right) = \ln\left(\frac{c}{A}\right) + \mathbb{E}_r[\Phi \circ Z]$
Aggregate Labor Cost Share (Average inverse markup rate)	$\frac{\mathbb{E}_1(\ell)}{\mathbb{E}_1(r)} = \mathbb{E}_r\left(\frac{1}{\mu}\right) = 1 - \left[\mathbb{E}_\pi\left(\frac{\mu}{\mu - 1}\right)\right]^{-1} = \frac{1}{\mathbb{E}_\ell(\mu)}$
Aggregate Profit Share (Average inverse price elasticity)	$\frac{\mathbb{E}_1(\pi)}{\mathbb{E}_1(r)} = \mathbb{E}_r\left(\frac{1}{\sigma}\right) = \frac{1}{\mathbb{E}_\pi(\sigma)} = 1 - \left[\mathbb{E}_\ell\left(\frac{\sigma}{\sigma - 1}\right)\right]^{-1}$

by applying the above formulae to $\pi(\cdot)/r(\cdot) = 1 - \ell(\cdot)/r(\cdot) = 1/\sigma(\cdot) = 1 - 1/\mu(\cdot)$,

Revisiting Melitz (2003) under CES: $s(z) = \gamma z^{1-\sigma}$

Pricing: $$\mu\left(\frac{\psi}{A}\right) = \frac{\sigma}{\sigma-1} > 1 \Rightarrow \rho\left(\frac{\psi}{A}\right) = 1$$

$$\Rightarrow \mathcal{E}_r\left(\frac{\psi}{A}\right) = \mathcal{E}_\pi\left(\frac{\psi}{A}\right) = \mathcal{E}_\ell\left(\frac{\psi}{A}\right) = 1-\sigma < 0.$$

Relative firm size, in revenue, profit, employment, unchanged across equilibriums.

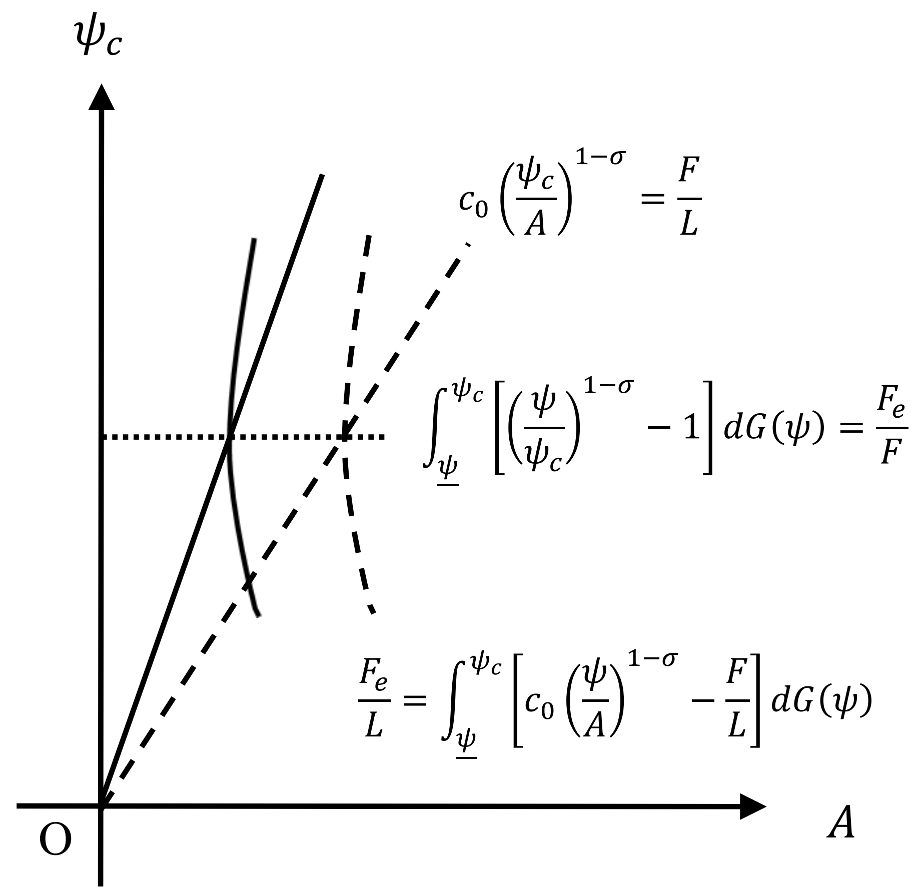
Cutoff Rule: $$c_0 L\left(\frac{\psi_c}{A}\right)^{1-\sigma} = F,$$

Free Entry Condition: $$\int_{\underline{\psi}}^{\psi_c}\left[c_0 L\left(\frac{\psi}{A}\right)^{1-\sigma} - F\right] dG(\psi) = F_e,$$

with $c_0 > 0$. As L changes, the intersection moves along

$$\int_{\underline{\psi}}^{\psi_c}\left[\left(\frac{\psi}{\psi_c}\right)^{1-\sigma} - 1\right] dG(\psi) = \frac{F_e}{F}$$

horizontal, i.e., independent of A, hence of L.



Proposition 1: Under CES,

- $L \uparrow$ keeps ψ_c unaffected; increases both M and $MG(\psi_c)$ *proportionately;* All adjustments at ***the extensive margin.***
- $F_e \downarrow$ reduces ψ_c; increases M; increases (decreases) $MG(\psi_c)$ if $\mathcal{E}_G'(\psi) < (>) 0$; $MG(\psi_c)$ unaffected under Pareto.
- $F \downarrow$ increases ψ_c; increases $MG(\psi_c)$; increases (decreases) M if $\mathcal{E}_G'(\psi) < (>) 0$; M unaffected under Pareto.

Cross-Sectional Implications under 2nd & 3rd Laws

Marshall's 2nd Law: Cross-Sectional Implications (Proposition 2)

(A2): $\zeta(z_\psi)$ is increasing in $z_\psi \equiv p_\psi/A = Z(\psi/A)$
Note: **A2 ⇒ A1.**

- **Price elasticity** $\zeta(Z(\psi/A)) \equiv \sigma(\psi/A)$, $\sigma'(\psi/A) > 0$; high-ψ firms operate at more elastic parts of demand curve.
  - **Markup Rate**, $\mu(\psi/A)$, decreasing in $\psi/A \Leftrightarrow \mathcal{E}_\mu(\psi/A) < 0$; high-$\psi$ firms charge lower markup rates.
  - **Incomplete Pass-Through:** The pass-through rate, $\rho(\psi/A) = 1 + \mathcal{E}_\mu(\psi/A) < 1$.

- **Procompetitive effect of entry/Strategic complementarity in pricing,** $\frac{\partial \ln p_\psi}{\partial \ln A} = 1 - \rho(\psi/A) = -\mathcal{E}_\mu(\psi/A) > 0$.
  Markups lower under more competitive pressures ($A = A(\mathbf{p}) \downarrow$), due to either a larger Ω and/or a lower $\mathbf{p}$

Lemma 5: For a positive-valued function of a single variable, $f(\cdot)$,

$$sgn\left\{\frac{\partial^2 \ln f(\psi/A)}{\partial\psi\partial A}\right\} = -sgn\left\{\mathcal{E}_f'\left(\frac{\psi}{A}\right)\right\} = -sgn\left\{\frac{d^2 \ln f\left(e^{\ln(\psi/A)}\right)}{(d\ln(\psi/A))^2}\right\}$$

$f(\psi/A)$ *log-super(sub)modular* in ψ & $A \Leftrightarrow \mathcal{E}_f'(\cdot) < (>)0 \Leftrightarrow \ln f\left(e^{\ln(\psi/A)}\right)$ concave (convex) in $\ln(\psi/A)$

- **Profit,** $\pi(\psi/A)L$, always decreasing, **strictly log-supermodular** in ψ and A.
  $A \downarrow$ → a proportionately larger decline in profit for high-ψ firms→ Larger dispersion of profit

3rd Law: Cross-Sectional Implications (Propositions 3, 4, and 5)

In addition to A2, if we further assume, with some empirical support, e.g. Berman et.al.(2012), Amiti et.al.(2019),

(A3): $\mathcal{E}'_{\zeta/(\zeta-1)}(z) \geq (>) 0 \Leftrightarrow \mathcal{E}'_{\mu}(\psi/A) = \rho'(\psi/A) \geq (>) 0$. --we call it **Weak (Strong) 3rd Law**.
Under translog, $\rho(\psi/A)$ is strictly decreasing, violating A3

- **Markup rate**, $\mu(\psi/A)$, decreasing under A2, **log-submodular** in ψ & A under A3. For strong A3, it is strict and $A\downarrow$ → a proportionately smaller decline in markup rate for high-ψ firms → smaller dispersion of markup rate

- **Revenue**, $r(\psi/A)L$, always decreasing, **strictly log-supermodular** in ψ & A under *weak A3*
  $A\downarrow$ → a proportionately larger decline in revenue for high-ψ firms → Larger dispersion of revenue

- **Employment,** $\ell(\psi/A)L = \frac{r(\psi/A)}{\mu(\psi/A)}L$, *hump-shaped* in ψ/A, **strictly log-supermodular** in ψ & A under *weak A3*
  Employment is increasing in ψ across all active firms with a large enough overhead/market size ratio.
  $A\downarrow$ → Employment up for the most productive firms.

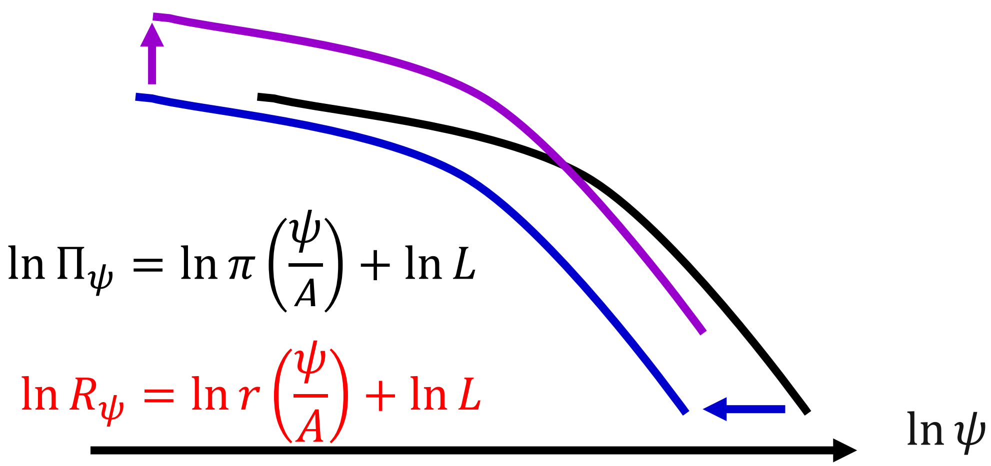
- **Pass-through rate,** $\rho(\psi/A)$, **strictly log-submodular** in ψ & A for a small enough $\bar{z}$ under strong A3
  $A\downarrow$ → a proportionately smaller increase in the pass-through rate for low-ψ firms among the active.

Cross-Sectional Implications of More Competitive Pressures, $A\downarrow$: A Graphic Representation

Profit(Revenue) Function: $\Pi_\psi = \pi(\psi/A)L$;, $R_\psi = r(\psi/A)L$

- *always* decreasing in ψ
- strictly log-supermodular *under A2 (Weak A3)*

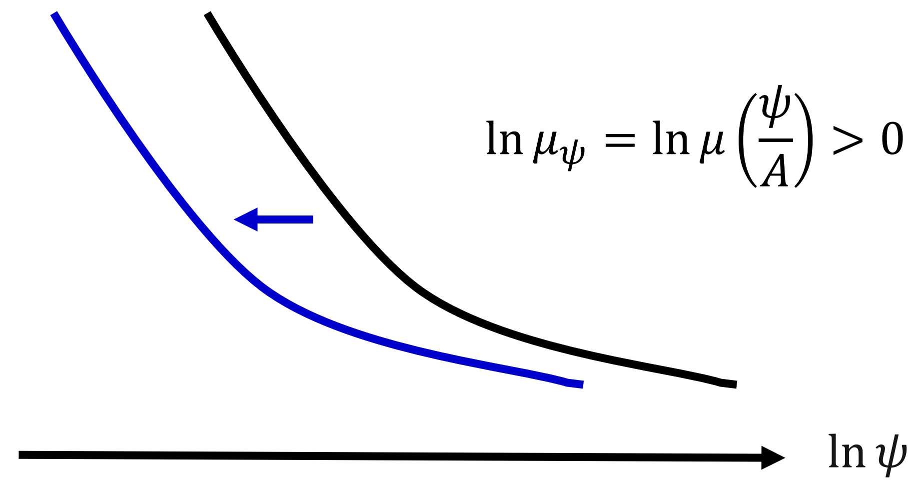
→ $A\downarrow$ with L fixed shifts down with a steeper slope at each ψ;
→ $A\downarrow$ due to $L\uparrow$, a parallel shift up, a single-crossing.



Markup Rate Function: $\mu_\psi = \mu(\psi/A) > 1$

- decreasing in ψ *under A2*
- weakly log-submodular *under Weak A3*
- strictly log-submodular *under Strong A3*

→ $A\downarrow$ shifts down with a flatter slope at each ψ

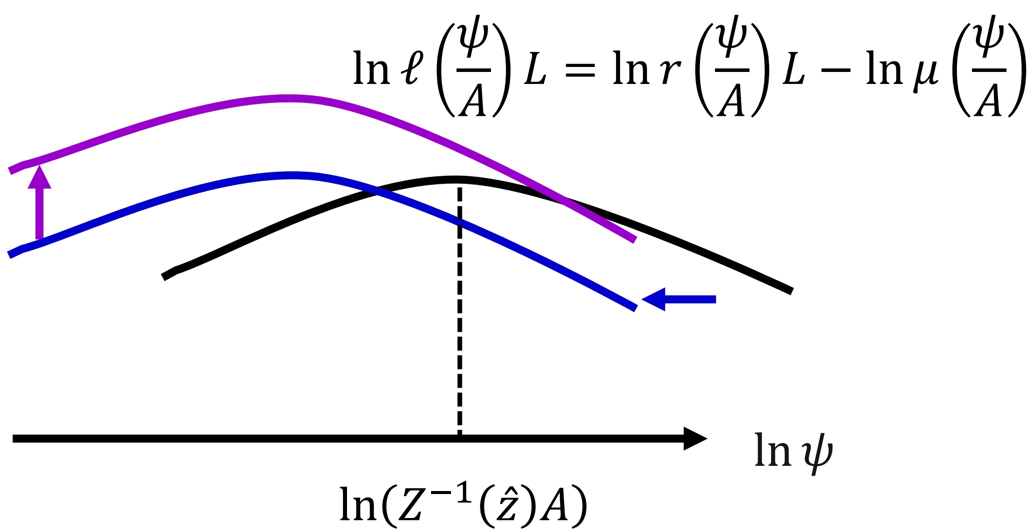


✓ With $\ln \psi$ in the horizontal axis, $A\downarrow$ causes a parallel leftward shift of the graphs in these figures.
✓ $f(\psi/A)$ is strictly log-super(sub)modular in ψ & $A \Leftrightarrow \ln f(\psi/A)$ is (strictly) concave(convex) in $\ln(\psi/A)$.

Employment Function: $\ell(\psi/A)L = r(\psi/A)L/\mu(\psi/A)$

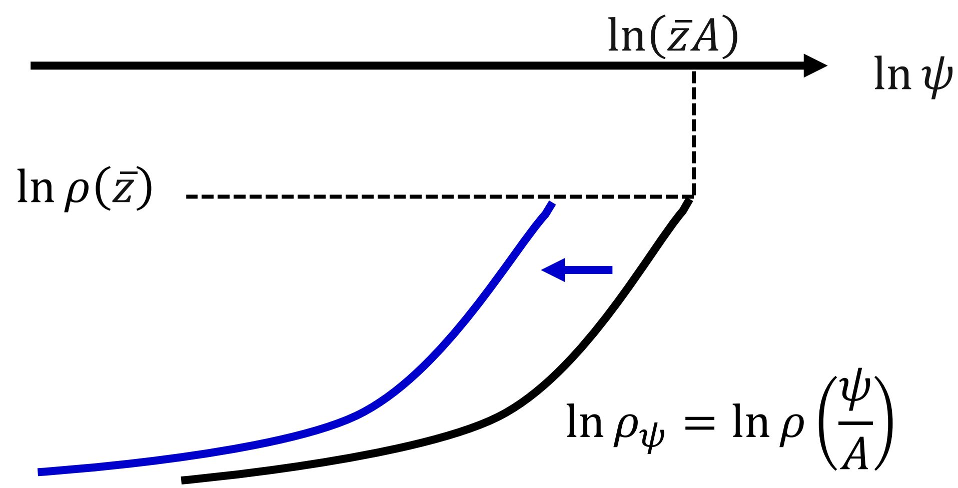
- *Hump-shaped* in ψ *under A2 and weak A3.*
  $\rightarrow A\downarrow$ shifts up (down) for a low (high) ψ with $A\downarrow$
- Strictly log-supermodular *under weak A3*
  for $A\downarrow$ with a fixed L; for $A\downarrow$ caused by $L\uparrow$

Single-crossing even with a fixed L



Pass-Through Rate Function: $\rho_\psi = \rho(\psi/A)$

- $\rho(\psi/A) < 1$ *under A2, hence it cannot be s*trictly log-submodular for a higher range of ψ/A
- Strictly increasing in ψ *under Strong A3*
- Strictly log-submodular for a lower range of ψ/A *under A2 and Strong A3* $\Rightarrow A\downarrow$ shifts up with a steeper slope at each ψ *with a small enough* $\bar{z}$*.*



In summary, more competitive pressures ($A\downarrow$)

- $\mu(\psi/A)\downarrow$ under A2 & $\rho(\psi/A)\uparrow$ under strong A3
- Profit, Revenue, Employment become more concentrated among the most productive.

Comparative Statics: General Equilibrium Effects

Comparative Statics: General Equilibrium Effects of F_e, L, and F on A and ψ_c

Proposition 6:

$$\begin{bmatrix} d\ln A \\ d\ln \psi_c \end{bmatrix} = \frac{\mathbb{E}_1(\pi)}{\mathbb{E}_1(\ell)} \begin{bmatrix} 1-f_x & f_x \\ 1-f_x & f_x-\delta \end{bmatrix} \begin{bmatrix} d\ln(F_e/L) \\ d\ln(F/L) \end{bmatrix}$$

where

$$\frac{\mathbb{E}_1(\pi)}{\mathbb{E}_1(\ell)} = \frac{1}{\mathbb{E}_\pi(\sigma)-1} = \{\mathbb{E}_r[\mu^{-1}]\}^{-1} - 1 = \mathbb{E}_\ell(\mu) - 1 > 0;$$

The average profit/average labor cost ratio among the active firms

$$f_x \equiv \frac{FG(\psi_c)}{F_e + FG(\psi_c)} = \frac{\pi(\psi_c/A)}{\mathbb{E}_1(\pi)} < 1;$$

The share of the overhead in the total expected fixed cost = to the profit of the cut-off firm relative to the average profit among the active firms

$$\delta \equiv \frac{\mathbb{E}_\pi(\sigma)-1}{\sigma(\psi_c/A)-1} = \frac{\pi(\psi_c/A)}{\ell(\psi_c/A)} \frac{\mathbb{E}_1(\ell)}{\mathbb{E}_1(\pi)} \equiv f_x \frac{\mathbb{E}_1(\ell)}{\ell(\psi_c/A)} > 0.$$

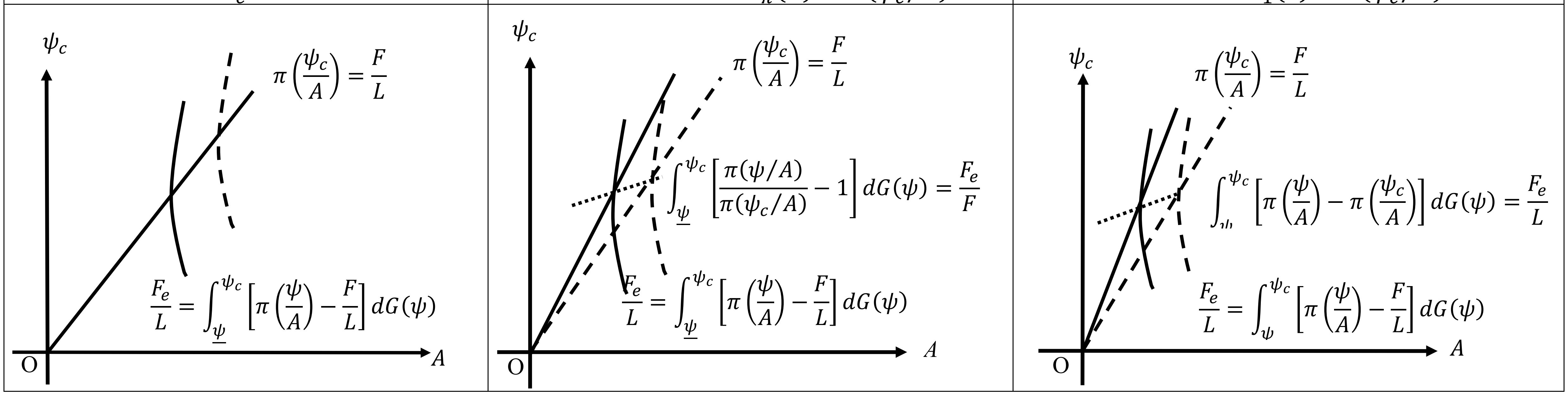
The profit/labor cost ratio of the cut-off firm to the average profit/average labor cost ratio among the active firms.

Corollary of Proposition 6

	A	ψ_c/A	ψ_c
F_e	$\frac{dA}{dF_e} > 0$	$\frac{d(\psi_c/A)}{dF_e} = 0$	$\frac{d\psi_c}{dF_e} > 0$
L	$\frac{dA}{dL} < 0$	$\frac{d(\psi_c/A)}{dL} > 0$	$\frac{d\psi_c}{dL} < 0 \Leftrightarrow \mathbb{E}_\pi(\sigma) < \sigma\left(\frac{\psi_c}{A}\right)$, which holds globally if $\sigma'(\cdot) > 0$, i.e., under A2
F	$\frac{dA}{dF} > 0$	$\frac{d(\psi_c/A)}{dF} < 0$	$\frac{d\psi_c}{dF} > 0 \Leftrightarrow \mathbb{E}_1(\ell) < \ell\left(\frac{\psi_c}{A}\right)$, which holds globally if $\ell'(\cdot) > 0$

.

$F_e \downarrow$	$L \uparrow$ under A2 $\Leftrightarrow \mathbb{E}_\pi(\sigma) < \sigma(\psi_c/A)$	$F \downarrow$ under $\mathbb{E}_1(\ell) < \ell(\psi_c/A)$



Note: For $F = 0$ & $\frac{\psi_c}{A} = \bar{z} < \infty$, the cutoff rule does not change $L \uparrow$ is isomorphic to $F_e \downarrow$

Market Size Effect on Profit and Revenue Distributions (Proposition 7)

7a: Under **A2,** there exists a unique $\psi_0 \in (\underline{\psi}, \psi_c)$ such that $\sigma\left(\frac{\psi_0}{A}\right) = \mathbb{E}_\pi(\sigma)$ with

$$\frac{d \ln \Pi_\psi}{d \ln L} > 0 \Leftrightarrow \sigma\left(\frac{\psi}{A}\right) < \mathbb{E}_\pi(\sigma) \text{ for } \psi \in (\underline{\psi}, \psi_0),$$

and

$$\frac{d \ln \Pi_\psi}{d \ln L} < 0 \Leftrightarrow \sigma\left(\frac{\psi}{A}\right) > \mathbb{E}_\pi(\sigma) \text{ for } \psi \in (\psi_0, \psi_c).$$

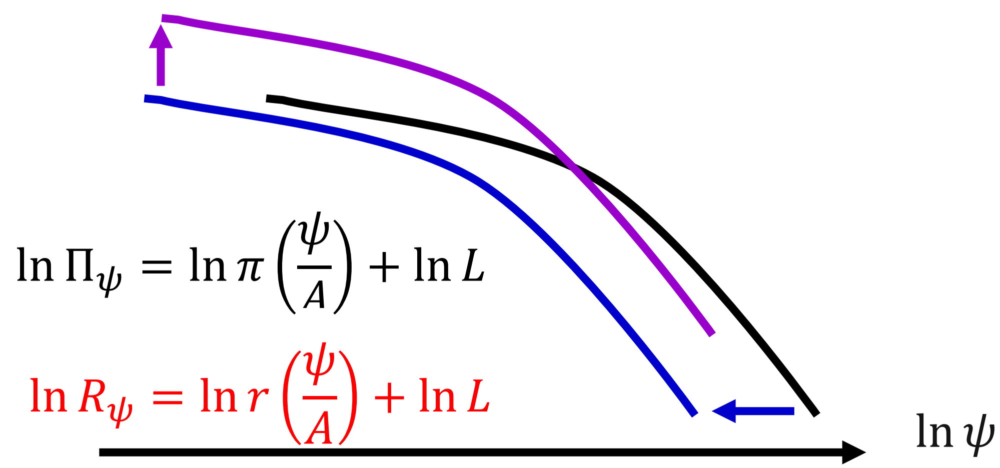
7b: Under **A2 and the weak A3,** there exists $\psi_1 > \psi_0$, such that

$$\frac{d \ln R_\psi}{d \ln L} > 0 \text{ for } \psi \in (\underline{\psi}, \psi_1).$$

Furthermore, $\psi_1 \in (\psi_0, \psi_c)$ and

$$\frac{d \ln R_\psi}{d \ln L} < 0 \text{ for } \psi \in (\psi_1, \psi_c),$$

for a sufficiently small F.



In short, more productive firms expand in absolute terms, while less productive firms shrink.

The Composition Effect: Average Markup and Pass-Through Rates

- Under A2, $A\downarrow$ causes $\mu(\psi/A)\downarrow$ for each ψ, but distribution shifts toward low-ψ firms with higher $\mu(\psi/A)$.
- Under strong A3, $A\downarrow$ causes $\rho(\psi/A)\uparrow$ for each ψ, but distribution shifts toward low-ψ firms with lower $\rho(\psi/A)$.

Proposition 8: *Assume that $\mathcal{E}_g'(\cdot)$ does not change its sign and $\underline{\psi}=0$. Consider a shock to F_e, L, and/or F, which affects competitive pressures, i.e., $dA\neq 0$. Then, the response of any weighted generalized mean of any monotone function, $f(\psi/A)>0$, defined by*

$$I\equiv\mathcal{M}^{-1}\left(\mathbb{E}_w\left(\mathcal{M}(f)\right)\right)$$

with a monotone transformation $\mathcal{M}:\mathbb{R}_+\to\mathbb{R}$ and a weighting function, $w(\psi/A)>0$, satisfies:

	$f'(\cdot)>0$	$f'(\cdot)=0$	$f'(\cdot)<0$
$\mathcal{E}_g'(\cdot)>0$	$\frac{d\ln(\psi_c/A)}{d\ln A}\geq 0\Rightarrow\frac{d\ln I}{d\ln A}>0$	$\frac{d\ln I}{d\ln A}=0$	$\frac{d\ln(\psi_c/A)}{d\ln A}\geq 0\Rightarrow\frac{d\ln I}{d\ln A}<0$
$\mathcal{E}_g'(\cdot)=0$ (Pareto)	$\frac{d\ln(\psi_c/A)}{d\ln A}\gtreqless 0\Leftrightarrow\frac{d\ln I}{d\ln A}\gtreqless 0$	$\frac{d\ln I}{d\ln A}=0$	$\frac{d\ln(\psi_c/A)}{d\ln A}\gtreqless 0\Leftrightarrow\frac{d\ln I}{d\ln A}\lesseqgtr 0$
$\mathcal{E}_g'(\cdot)<0$	$\frac{d\ln(\psi_c/A)}{d\ln A}\leq 0\Rightarrow\frac{d\ln I}{d\ln A}<0$	$\frac{d\ln I}{d\ln A}=0$	$\frac{d\ln(\psi_c/A)}{d\ln A}\leq 0\Rightarrow\frac{d\ln I}{d\ln A}>0$

Moreover, if $\mathcal{E}_g'(\cdot)=\frac{d\ln(\psi_c/A)}{d\ln A}=0$, $d\ln I/d\ln A=0$ for any $f(\psi/A)$, monotonic or not. Furthermore, $\mathcal{E}_g'(\cdot)$ can be replaced with $\mathcal{E}_G'(\cdot)$ in all the above statements for $w(\psi/A)=1$, i.e., the unweighted averages.

The arithmetic, $I=\left(\mathbb{E}_w(f)\right)$, geometric, $I=\exp[\mathbb{E}_w(\ln f)]$, harmonic, $I=\left(\mathbb{E}_w(f^{-1})\right)^{-1}$, means are special cases.
The weight function, $w(\psi/A)$, can be profit, revenue, and employment.

Corollary 1 of Proposition 8

a) Entry Cost: $f'(\cdot)\mathcal{E}_g'(\cdot) \gtreqless 0 \Leftrightarrow \frac{d\ln I}{d\ln F_e} = \frac{d\ln I}{d\ln A}\frac{d\ln A}{d\ln F_e} \gtreqless 0.$

b) Market Size: *If* $\mathcal{E}_g'(\cdot) \leq 0$, *then,* $f'(\cdot) \gtreqless 0 \Longrightarrow \frac{d\ln I}{d\ln L} = \frac{d\ln I}{d\ln A}\frac{d\ln A}{d\ln L} \gtreqless 0.$

c) Overhead Cost: *If* $\mathcal{E}_g'(\cdot) \leq 0$, *then,* $f'(\cdot) \gtreqless 0 \Longrightarrow \frac{d\ln I}{d\ln F} = \frac{d\ln I}{d\ln A}\frac{d\ln A}{d\ln F} \lesseqgtr 0.$

Furthermore, $\mathcal{E}_g'(\cdot)$ *can be replaced with* $\mathcal{E}_G'(\cdot)$ *for* $w(\psi/A) = 1$, *i.e., the unweighted averages.*

For the entry cost, $\frac{d\ln(\psi_c/A)}{d\ln A} = 0$.

- $\mathcal{E}_g'(\cdot) > 0$; sufficient & necessary for the composition effect to dominate:
    - The average markup & pass-through rates move in the *opposite* direction from the firm-level rates
- $\mathcal{E}_g'(\cdot) = 0$ (Pareto); a knife-edge. $A \downarrow\rightarrow$ no change in average markup and pass-through.
- $\mathcal{E}_g'(\cdot) < 0$; sufficient & necessary for the procompetitive effect to dominate:
  The average markup & pass-through rates move in the *same* direction from the firm-level rates

For market size and the overhead cost, $\frac{d\ln(\psi_c/A)}{d\ln A} < 0$

- $\mathcal{E}_g'(\cdot) > 0$; necessary for the composition effect to dominate:
- $\mathcal{E}_g'(\cdot) \leq 0$; sufficient for the procompetitive effect to dominate:

The Composition Effect: Impact on P/A

$$\ln\left(\frac{A}{cP}\right) = \mathbb{E}_r[\Phi \circ Z]$$

$$\zeta'(\cdot) \gtreqless 0 \;\Longrightarrow \Phi'(\cdot) \lesseqgtr 0 \Leftrightarrow \Phi \circ Z'(\cdot) \lesseqgtr 0$$

Corollary 2 of Proposition 8: *Assume $\underline{\psi} = 0$, and neither $\zeta'(\cdot)$ nor $\mathcal{E}_g'(\cdot)$ change the signs. Consider a shock to F_e, L, and/or F, which affects competitive pressures, i.e., $dA \neq 0$. Then, the response of P/A satisfies:*

	$\zeta'(\cdot) > 0$ (A2)	$\zeta'(\cdot) = 0$ (CES)	$\zeta'(\cdot) < 0$
$\mathcal{E}_g'(\cdot) > 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \geq 0 \Longrightarrow \frac{d\ln(P/A)}{d\ln A} > 0$	$\frac{d\ln(P/A)}{d\ln A} = 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \geq 0 \Longrightarrow \frac{d\ln(P/A)}{d\ln A} < 0$
$\mathcal{E}_g'(\cdot) = 0$ (Pareto)	$\frac{d\ln(\psi_c/A)}{d\ln A} \gtreqless 0 \Leftrightarrow \frac{d\ln(P/A)}{d\ln A} \gtreqless 0$	$\frac{d\ln(P/A)}{d\ln A} = 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \gtreqless 0 \Leftrightarrow \frac{d\ln(P/A)}{d\ln A} \lesseqgtr 0$
$\mathcal{E}_g'(\cdot) < 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \leq 0 \Longrightarrow \frac{d\ln(P/A)}{d\ln A} < 0$	$\frac{d\ln(P/A)}{d\ln A} = 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \leq 0 \Longrightarrow \frac{d\ln(P/A)}{d\ln A} > 0$

Comparative Statics on $MG(\psi_c)$

Proposition 9: *Assume that $\mathcal{E}_G'(\cdot)$ does not change its sign and $\underline{\psi} = 0$. Consider a shock to F_e, F, and/or L, which affects competitive pressures, i.e., $dA \neq 0$. Then, the response of the mass of active firms, $MG(\psi_c)$, is as follows:*

$$If\ \mathcal{E}_G'(\cdot) > 0, \qquad \frac{d\ln(\psi_c/A)}{d\ln A} \geq 0 \Rightarrow \frac{d\ln[MG(\psi_c)]}{d\ln A} > 0;$$

$$If\ \mathcal{E}_G'(\cdot) = 0, \qquad \frac{d\ln(\psi_c/A)}{d\ln A} \gtreqless 0 \Leftrightarrow \frac{d\ln[MG(\psi_c)]}{d\ln A} \gtreqless 0;$$

$$If\ \mathcal{E}_G'(\cdot) < 0, \qquad \frac{d\ln(\psi_c/A)}{d\ln A} \leq 0 \Rightarrow \frac{d\ln[MG(\psi_c)]}{d\ln A} < 0.$$

Corollary 1 of Proposition 9

a) Entry Cost: $\mathcal{E}_G'(\cdot) \gtreqless 0 \Leftrightarrow \frac{d\ln[MG(\psi_c)]}{d\ln F_e} = \frac{d\ln[MG(\psi_c)]}{d\ln A}\frac{d\ln A}{d\ln F_e} \gtreqless 0.$

b) Market Size: $\mathcal{E}_G'(\cdot) \leq 0 \Rightarrow \frac{d\ln[MG(\psi_c)]}{d\ln L} = \frac{d\ln[MG(\psi_c)]}{d\ln A}\frac{d\ln A}{d\ln L} > 0.$

c) Overhead Cost: $\mathcal{E}_G'(\cdot) \leq 0 \Rightarrow \frac{d\ln[MG(\psi_c)]}{d\ln F} = \frac{d\ln[MG(\psi_c)]}{d\ln A}\frac{d\ln A}{d\ln F} < 0.$

For a decline in the entry cost,
$\mathcal{E}_g'(\cdot) > 0$ *sufficient & necessary* for $MG(\psi_c)\downarrow$; $\mathcal{E}_g'(\cdot) = 0$, no effect; $\mathcal{E}_g'(\cdot) < 0$; *sufficient & necessary* for $MG(\psi_c)\uparrow$
For market size and the overhead cost
$\mathcal{E}_g'(\cdot) > 0$ *necessary* for $MG(\psi_c)\downarrow$; $\mathcal{E}_g'(\cdot) \leq 0$ *sufficient* for $MG(\psi_c)\uparrow$

Impact of Competitive Pressures on Unit Cost/TFP

By combining Corollary 2 of Proposition 8 and Corollary 1 of Proposition,

Corollary 2 of Proposition 9: *Assume* $\underline{\psi} = 0$, *and neither* $\zeta'(\cdot)$ *nor* $\mathcal{E}_g'(\cdot)$ *change the signs. Consider a shock to* F_e, L, *and/or* F, *which affects competitive pressures, i.e.,* $dA \neq 0$. *Then, the response of* P *satisfies:*

	$\zeta'(\cdot) > 0$ (A2)	$\zeta'(\cdot) = 0$ (CES)	$\zeta'(\cdot) < 0$
$\mathcal{E}_g'(\cdot) > 0$	$\frac{d \ln P}{d \ln A} > 1 \; for \; F_e$	$\frac{d \ln P}{d \ln A} = 1$	?
$\mathcal{E}_g'(\cdot) = 0$ (Pareto)	$\frac{d \ln P}{d \ln A} = 1 \; for \; F_e$ $0 < \frac{d \ln P}{d \ln A} < 1 \; for \; F \; or \; L;$	$\frac{d \ln P}{d \ln A} = 1$	$\frac{d \ln P}{d \ln A} = 1 \; for \; F_e$ $\frac{d \ln P}{d \ln A} > 1 \; for \; F \; or \; L$
$\mathcal{E}_g'(\cdot) < 0$	$0 < \frac{d \ln P}{d \ln A} < 1$	$\frac{d \ln P}{d \ln A} = 1$	$\frac{d \ln P}{d \ln A} > 1$

Sorting of Heterogenous Firms: A Multi-Market Setting

Sorting: GE Implications in a Multi-Market Setting

Many markets of different size. Firms, after learning their ψ, choose which market to enter.

Proposition 10: Assortative Matching

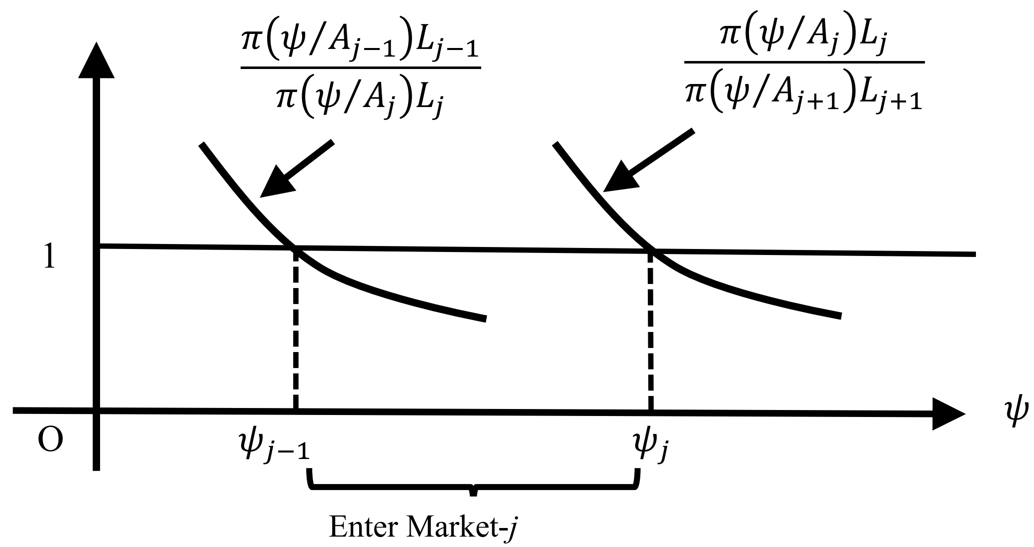
More competitive pressures in larger markets:

$$L_1 > L_2 > \cdots > L_J > 0 \;\Rightarrow\; 0 < A_1 < A_2 < \cdots < A_J < \infty$$

Under A2, more efficient firms sort themselves into larger markets: Firms $\psi \in (\psi_{j-1}, \psi_j)$ entering market-j, where

$$0 \leq \underline{\psi} = \psi_0 < \psi_1 < \psi_2 < \cdots < \psi_J < \overline{\psi} \leq \infty.$$

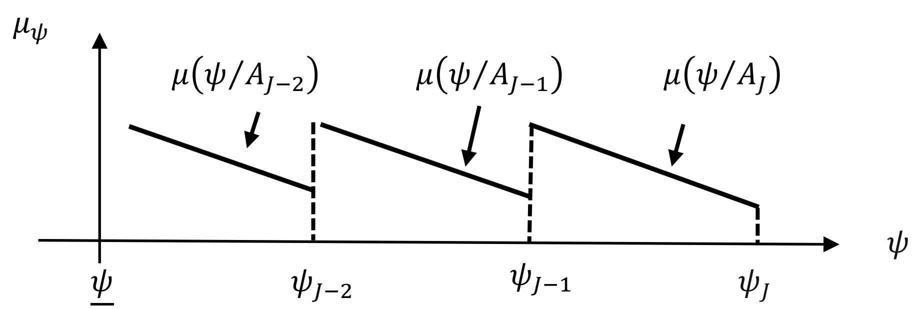
Sorting: GE Implications in a Multi-Market Setting



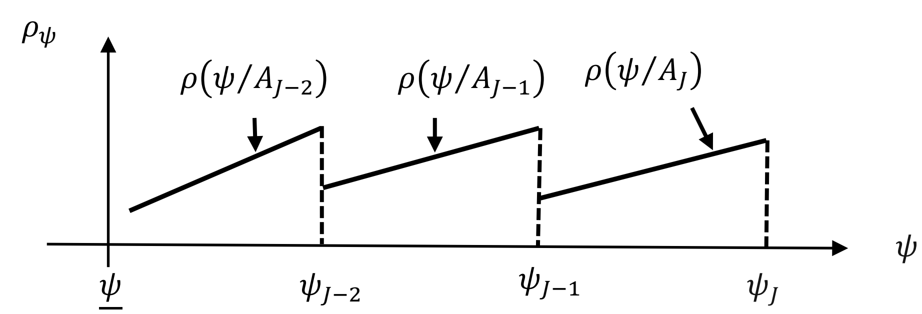
Proposition 11: The Composition Effect: *Examples* with Pareto-productivity such that

- The average markup rates *higher* (the average pass-through rates *lower* under Strong A3) in larger (more competitive) markets
- A decline in F_e causes uniform declines in ψ_j & A_j with the average markup/pass-through rates unchanged.

Markup Rate across markets under A2



Pass-Through Rate across markets under strong A3



A caution against testing A2/A3 by comparing the average markup/pass-through rates in cross-section of cities.

Three Parametric Families of H.S.A. (Appendix D)

Generalized Translog For $\eta > 0, \sigma > 1$	$s(z) = \gamma\left(-\frac{\sigma-1}{\eta}\ln\left(\frac{z}{\bar{z}}\right)\right)^{\eta}$; $z < \bar{z} \equiv \beta e^{\frac{\eta}{\sigma-1}}$	$1 - \frac{1}{\zeta(z)} = \frac{\eta}{\eta - \ln\left(\frac{z}{\bar{z}}\right)} \Rightarrow \begin{matrix} \mathcal{E}_\mu(\cdot) < 0 \\ \mathcal{E}'_\mu(\cdot) = \rho'(\cdot) < 0 \end{matrix}$ satisfying **A2;** violating **A3.**

Translog is the special case where $\eta = 1$. CES is the limit case, as $\eta \to \infty$, while holding $\beta > 0$ and $\sigma > 1$ fixed.

Constant Pass-Through (CoPaTh) For $0 < \rho < 1, \sigma > 1$	$s(z) = \gamma\sigma^{\frac{\rho}{1-\rho}}\left[1 - \left(\frac{z}{\bar{z}}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}}$; $\bar{z} \equiv \beta\left(\frac{\sigma}{\sigma-1}\right)^{\frac{\rho}{1-\rho}}$	$1 - \frac{1}{\zeta(z)} = \left(\frac{z}{\bar{z}}\right)^{\frac{1-\rho}{\rho}} \Rightarrow \begin{matrix} \mathcal{E}_\mu(\cdot) < 0 \\ \mathcal{E}'_\mu(\cdot) = \rho'(\cdot) = 0 \end{matrix}$; satisfying **A2** & weak **A3;** violating strong **A3**

CES is the limit case, as $\rho \to 1$, while holding $\beta > 0$ and $\sigma > 1$ fixed.

Power Elasticity of Markup Rate (Fréchet Inverse Markup Rate) For $\kappa \geq 0$ and $\lambda > 0$	$s(z) = \exp\left[\int_{z_0}^{z} \frac{c}{c - \exp\left[-\frac{\kappa\bar{z}^{-\lambda}}{\lambda}\right]\exp\left[\frac{\kappa\xi^{-\lambda}}{\lambda}\right]} \frac{d\xi}{\xi}\right]$	$1 - \frac{1}{\zeta(z)} = c\exp\left[\frac{\kappa\bar{z}^{-\lambda}}{\lambda}\right]\exp\left[-\frac{\kappa z^{-\lambda}}{\lambda}\right]$ $\Rightarrow \mathcal{E}_\mu(\cdot) < 0; \mathcal{E}'_\mu(\cdot) = \rho'(\cdot) > 0$ satisfying **A2** and strong **A3** for $\kappa > 0$ and $\lambda > 0$.

CES for $\kappa = 0$; $\bar{z} = \infty$; $c = 1 - \frac{1}{\sigma}$; CoPaTh for $\bar{z} < \infty$; $c = 1$; $\kappa = \frac{1-\rho}{\rho} > 0$, and $\lambda \to 0$.